Exercise 14

A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s. Find the rate at which the area within the circle is increasing after (a) 1 s, (b) 3 s, and (c) 5 s. What can you conclude?

Solution

The area of a circle is
$$A(r) = \pi r^2.$$
Take the derivative with respect to time by using the chain rule.
$$\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt} = (2\pi r)\frac{dr}{dt}$$
Note that the radius (in centimeters) after t seconds have passed is $r(t) = 60t$, and the rate at which the radius is increasing (in centimeters per second) is $dr/dt = 60$.
$$\frac{dA}{dt} = 2\pi(60t)(60) = 7200\pi t$$
The rate at which the area is increasing after 1 s is
$$\left.\frac{dA}{dt}\right|_{t=1} = 7200\pi(1) = 7200\pi \text{ cm}^2/\text{s}.$$
The rate at which the area is increasing after 3 s is
$$\left.\frac{dA}{dt}\right|_{t=3} = 7200\pi(3) = 21\,600\pi \text{ cm}^2/\text{s}.$$
The rate at which the area is increasing after 5 s is
$$\left.\frac{dA}{dt}\right|_{t=3} = 7200\pi(5) = 36\,000\pi \text{ cm}^2/\text{s}.$$
Notice that the rate of increase of the area increases linearly in t.